South African Reserve Bank Working Paper Series WP/22/16

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13 December 2022

SOUTH AFRICAN RESERVE BANK

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Optimal interest rate gaps for flexible inflation targeting

Eric Schaling[*] and Kgotso Morema[†]

Abstract

The purpose of this paper is to build a small macro model along the lines of the SARB's Quarterly Projection Model (QPM). We infer optimal monetary policy rules in terms of interest rate gaps (as defined by the QPM). We investigate flexible inflation targeting augmented with interest rate smoothing and the zero lower bound. We find that between 2000 and 2002 the actual repo rate was below both the strict output gap targeting implied repo rate and the strict inflation targeting implied repo rate. This implies that monetary policy was too accommodative. Similarly, between 2006 and early 2009 (before the global financial crisis), monetary policy appears to have been too accommodative. Interestingly, we find no evidence of periods when monetary policy was too restrictive. We also find that in the case of flexible inflation targeting – since the adoption of the QPM in 2017 – policymakers seem to have placed a bigger weight on inflation than growth.

JEL classification: B22, C01, C54, C61

Keywords: monetary policy, optimisation, monetary policy rules

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1. Introduction[1]

The purpose of this paper is to build a small macro model that combines parts of the SARB's Quarterly Projection Model (QPM) (Botha et al. 2017) with relevant other empirical work on neutral real interest rate (Kuhn, Ruch and Steinbach 2019) and short-term supply shocks (Botha, Ruch and Steinbach 2018). Our model would typically be closed with a monetary policy Taylor-type instrument rule employed in structural models such as the SARB's QPM (and models used at other inflation-targeting central banks, for example as discussed in Benes et al. (2016), Szilágyi et al. (2013) and Kamber et al. (2015)).

As concisely explained in Svensson (2004), an instrument rule is a formula for setting the central bank's instrument rate as a given function of observable variables. The best-known example of a simple instrument rule is the Taylor rule (Taylor 1993), where the instrument rate is a linear function of the inflation gap (the difference between inflation and the inflation target) and the output gap (the deviation of actual GDP from potential/equilibrium output). The problem with instrument rules is that they are ad hoc and there is no clear link between optimally implementing (flexible) inflation targeting and the instrument settings.[2] In contrast, a targeting rule specifies a condition to be fulfilled by the central bank's target variables (or forecasts thereof). For example, Svensson (1997) shows that strict inflation targeting (per the central bank's inflation-targeting regime) requires the two-year ahead inflation forecast to be equal to the inflation target (because of lags in the transmission mechanism). From this first-order condition, the 'correct' level of the short-term interest rate – as opposed to the ad-hoc one from the Taylor instrument rule – can then be inferred. The procedure is typically as follows. First, we use a loss function that penalises deviations of inflation from the target and output from potential output (that is, a non-zero output gap) and minimise that loss function subject to the model of the economy. The latter is the key objective of this paper. We will infer optimal monetary policy rules in terms of interest rate gaps (as defined according to the QPM) – and hence repo rates – that implement flexible inflation targeting augmented with interest rate smoothing and the zero lower bound.

[1] We are grateful to Shaun de Jager, seminar participants at the SARB and two anonymous referees for their comments and suggestions.

[2] In other words, we have a targeting rule that follows from optimisation. In contrast, an instrument rule will typically not minimise the loss function, hence we call it ad hoc.

Thus, we will minimise a plausible objective function subject to our calibrated small macro model. This has not been done for South Africa. The outcomes can be compared with actual repo rate setting across the sample, to serve as benchmarks and infer apparent policy preferences. For example, we will be able to look at distinct sub-cases of inflation targeting. What would the interest rate path look like if the central bank only cared about the output gap? Conversely, what repo rates are consistent with strict inflation hawk preferences? Of course, this investigation is general. The policy preferences – inflation versus output gap versus interest rate inertia (the latter postulated according to the instrument rule used in the QPM) – can be freely chosen and their implications can be traced for the alternative interest rate paths.

The remainder of the paper is organised as follows. In section 1.1 we survey related literature. Section 2 introduces the model. The targeting rules are discussed in section 3. Section 4 presents our empirical results. Appendix A shows all the data used as inputs in this paper. Variable names and naming conventions are presented in Appendix B. The derivation of the rational expectations solution is explained in Appendix C.

1.1 Literature review

In this section we provide a brief survey of work on optimal monetary policy rules. We start with work that uses ad-hoc models and then discuss papers that use the New Keynesian framework featuring micro foundations as implied by the general equilibrium approach.

In 1993, John Taylor introduced an instrument rule linking the United States Federal Reserve’s interest rate to levels of inflation and the output gap (Taylor 1993). The real (ex-post) federal funds rate is assumed to move up or down by half the difference between inflation and the inflation target. Each percentage point of the output gap also moves the federal funds rate by half a percentage point. It is important to point out that this rule does not take the zero lower bound into account.

Svensson (1997) discusses the difference between instrument rules – such as the Taylor rule proposed by Taylor – and optimal monetary policy rules, or what are called targeting rules. He uses a model, predating the New Keynesian approach, that does not have rational expectations but includes lags in the transmission mechanism of monetary policy. More specifically, the repo rate affects the output gap with a one period lag and the output gap affects inflation with a one period lag. Then inflation targeting – as implied by optimality – leads to inflation forecast targeting: the central bank's inflation forecast becomes an explicit intermediate target. The weight on output stabilisation determines how quickly the inflation forecast is adjusted towards the inflation target. □

Clarida, Gali and Gertler (1999) discuss the monetary policy design problem[3] within a New Keynesian framework – with forward-looking inflation and output gap expectations – consistent with micro foundations such as the consumption Euler equation and optimal price-setting by firms. Among other things, they show that the optimal monetary policy implicitly incorporates inflation targeting. They also consider the implications of frictions such as imperfect information. These frictions are discussed in more detail by Svensson and Woodford (2003). They investigate optimal monetary policy in a New Keynesian model (with forward-looking expectations) with partially observable potential output both for equilibria under discretion and under commitment.

Adam and Billi (2004) determine optimal discretionary monetary policy in a New Keynesian model when nominal interest rates are bounded below by zero. They find that nominal interest rates should be lowered faster in response to adverse shocks than in the case without this bound. According to their analysis, proactive lowering of interest rates is optimal because expectations of a possibly binding zero lower bound in the future amplify the effects of adverse shocks. Losses increase further when inflation is partly determined by lagged inflation in the Phillips curve.

Using a New Keynesian model, Davig and Guerkanyak (2015) argue that the real world is characterised by multiple inefficiencies and policymakers who have various objectives. The one policy tool that a central bank has control over – short-term nominal interest rates – will not undo all inefficiencies. They stress that asking policy questions only in terms of optimal monetary policy effectively gives other policymakers – such as

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3 The authors characterise this problem as how interest rates should adjust to the current state of the economy, should the central bank have a formal commitment to a policy rule or not.

the fiscal authority – a free hand to pursue their own goals. This worsens the trade-offs faced by the central bank. The literature on optimal monetary policy and the optimal simple rules – such as flexible inflation targeting as discussed by Svensson (1997) – assign all cyclical policymaking duties to central banks. This distorts the policy discussion and narrows monetary policy rules to a sub-optimal set.

Sims and Wu (2019) develop a New Keynesian model featuring financial intermediation, short- and long-term bonds, credit shocks, and scope for unconventional monetary policy or quantitative easing (QE). Credit shocks and QE appear in both the investment savings (IS) and Phillips curves. Optimal monetary policy entails adjusting the short-term interest rate to offset natural rate shocks and using QE to offset credit market disruptions. The ability of the central bank to engage in QE significantly mitigates the costs of a binding zero lower bound.

This paper uses a hybrid New Keynesian model with forward and backward terms for the output gap in the IS curve and survey inflation expectations in the Phillips curve, in line with the SARB's QPM model (Botha et al. 2017). We study the model under discretion – that is, given private sector inflation expectations – and incorporating the zero lower bound. Commitment – where inflation expectations are internalised to the central bank's optimisation problem – cannot be studied as inflation expectations are not rational.[4]

2. The model

Before discussing the key equations of the model, it is worth explaining the notation that we have used. Let x_t represent any variable (in terms of log x 100). Where a hat appears above the variable, $\hat{x}_t$, it indicates that the variable is a gap, or deviation from equilibrium.[5] When a line appears above the variable, $\bar{x}_t$, it refers to the variable's equilibrium value. To indicate the steady state of a variable, a superscript *ss* is used

4 In the case of commitment, the central bank takes into account how its actions affect inflation expectations.

5 The only exception is the definition of the structural parameters of the model – for example in the IS curve – as opposed to the reduced form parameters which have no hat.

and the time subscript *t* is not shown (since steady states remain constant over time). Finally, the quarter-on-quarter change in a variable is denoted by a Δ that precedes it: Δx_t. The only exception here is inflation – the quarter-on-quarter change in the (log) price level – where we use π, as it is the custom in most of the literature.

2.1 Block 1: the IS curve from the QPM

What follows in this section is based on the QPM (see Botha et al. 2017). The real economy is modelled as an IS curve. Here, the output gap is determined by four fundamental variables: (1) the real interest rate gap $\hat{r}_t$; (2) the real exchange rate gap $\hat{lz}_t$; (3) the foreign output gap $\hat{y}_t^F$; and (4) the deviation of South Africa's real export commodity price index from its trend value $\hat{tot}_t$. In addition, expectations about the future real economy, $E_t \hat{y}_{t+1}$, as well as recent outcomes, $\hat{y}_{t-1}$, matter for the dynamic behaviour of the output gap. Finally, the residual term $\hat{\varepsilon}_t^y$ captures demand shocks, while *a* represents the parameters that indicate the direct impact each of these variables have on the output gap.

Accordingly, in line with the SARB QPM, we have

$$\hat{y}_t = \hat{a}_1 E_t \hat{y}_{t+1} + \hat{a}_2 \hat{y}_{t-1} - \hat{a}_3 \hat{r}_t + \hat{a}_4 \hat{lz}_t + \hat{a}_5 \hat{y}_t^F + \hat{a}_6 \hat{tot}_t + \hat{\varepsilon}_t^y$$

or

$$\hat{y}_t = \hat{a}_1 E_t \hat{y}_{t+1} + \hat{a}_2 \hat{y}_{t-1} + h_t \qquad (2.1)$$

where $h_t \equiv -\hat{a}_3 \hat{r}_t + \hat{a}_4 \hat{lz}_t + \hat{a}_5 \hat{y}^F + \hat{a}_6 \hat{tot}_t + \hat{\varepsilon}_t^y$

We calibrate it as follows (again, according to the QPM):

$$\hat{a}_1 = 0.15, \ \hat{a}_2 = 0.60, \hat{a}_3 = 0.1425, \hat{a}_4 = 0.0075, \hat{a}_5 = 0.15, \ \hat{a}_6 = 0.01$$

where the output gap is defined as the difference between the log of real GDP and that of potential GDP (here indicated by the superscript), times 100, in order to express the gap in percentage terms:

$$\hat{y}_t = \left[\log(GDP_t - \log(GDP^{*})_t)\right] * 100 = y_t - \overline{y}_t$$

Apart from $\hat{r}_t$, the other variables, if positive, increase excess demand and hence inflation via the Phillips curve (see below). So, they are the demand-pull factors. The interest rate gap is defined as:

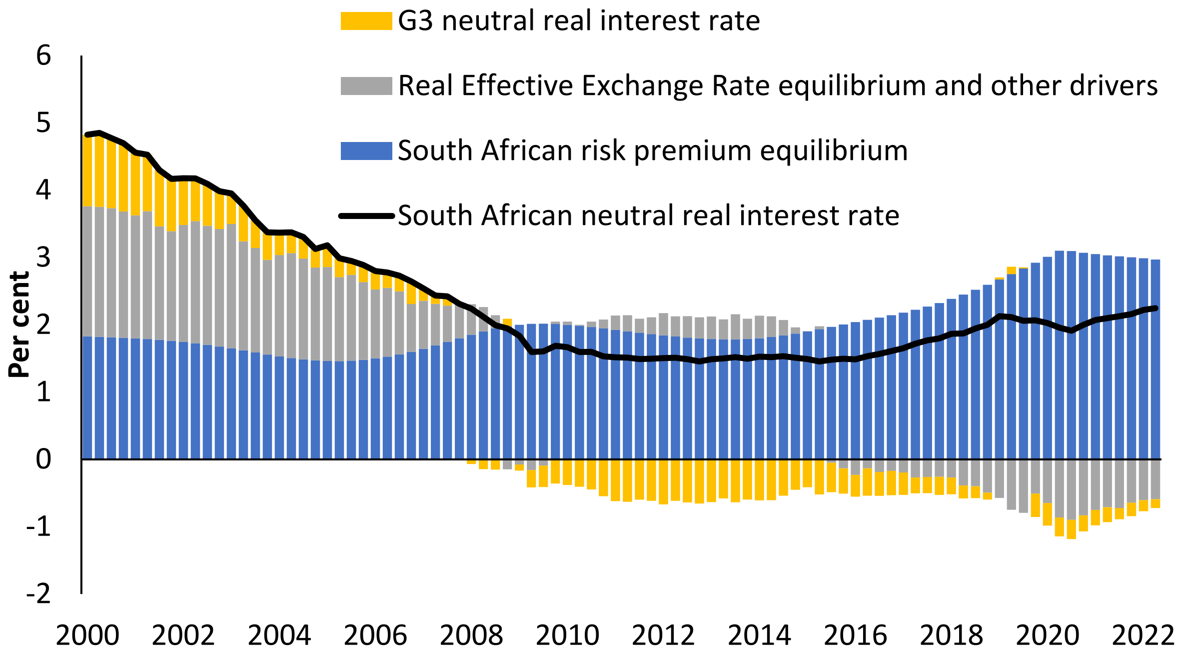
$$\hat{r}_t = r_t - \overline{r}_t \qquad (2.2)$$

and the nominal short-term interest rate is defined as:

$$i_t = r_t + E_t \pi_{t+1} \qquad (2.3)$$

The domestic natural real interest rate, NRIR, is estimated using a ‘rearranged’ equilibrium uncovered interest rate parity (UIP) condition, consistent with the SARB’s QPM (Botha et al. 2017). It is simply the sum of foreign NRIR, the expected change in the equilibrium real exchange rate and the equilibrium risk premium. Figure 1 shows the evolution of the domestic NRIR and its drivers. The major driver of the domestic NRIR is the country risk premium as South Africa is an emerging market, typically faced with high borrowing costs and requiring a higher domestic NRIR in equilibrium (Ruch and Steinbach 2017).

Figure 1: South Africa's real interest rate and its drivers



Source: SARB

2.2 Block 2: the Phillips curve

As in the QPM, we have a hybrid version of the New Keynesian Phillips curve which expresses current inflation as a function of the inflation rate that price setters expect to hold in the future, past inflation outcomes and price pressures that emanate from changes in the real costs of production – commonly referred to as real marginal costs.[6] In the model, these real marginal costs are generally driven by real wages, the real exchange rate and the output gap.[7] More specifically, the variables are: (1) the output gap $\hat{y}_t$; (2) inflation expectations $E_t\pi_{t+2}$; (3) lagged inflation π_{t-1}; (4) foreign (imported) inflation π_t^F, the real unit labour cost gap $\hat{rulc}_t$ and the deviation of the real exchange rate gap from equilibrium $\hat{lz}_t$. Finally, the residual term, $\hat{\varepsilon}_t^{\pi}$ captures

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6 The pure theoretical New Keynesian Phillips curve abstracts from past inflation outcomes. However, to match the empirical persistence of actual inflation, indexing to past inflation outcomes is added to the specification.

7 Real marginal costs are essentially the increase in production costs (in real terms) that are related to a marginal increase in output at the firm level.

supply shocks, while *b* represents the parameters that indicate the direct impact each of the above variables has on the inflation rate.[8]

Accordingly, the Phillips curve is given by

$$\pi_t = b_1 \hat{y}_t + b_2 E_t \pi_{t+2} + b_3 \pi_{t-1} + b_4 \pi_t^F + b_5 \hat{rulc}_t + b_6 \hat{lz}_t + \varepsilon_t^\pi \qquad (2.4)$$

We calibrate it as follows:

$$b_1 = 0.25,\ b_2 = 0.2, b_3 = 0.70, b_4 = 0.05, b_5 = 0.07, b_6 = 0.10$$

or

$$\pi_t = b_1 \hat{y}_t + b_3 \pi_{t-1} + j_t \qquad (2.5)$$

where $j_t \equiv b_2 E_t \pi_{t+2} + b_3 \pi_{t-1} + b_4 \pi_t^F + b_5 \hat{rulc}_t + b_6 \hat{lz}_t + \varepsilon_t^\pi$ represents potential 'cost-push' factors – as $\hat{y}_t$ captures demand pull.

3. Solution for the output gap – QPM with rational expectations

We now turn to the standard case of rational equations. It turns out that equation (2.1) is an expectational difference equation. Before we solve it, we must investigate if we have a unique equilibrium. In practice, this involves finding out whether the so-called Blanchard and Kahn (1980) conditions are satisfied. In Appendix C, we show that this is the case for our calibration. Having checked for stability we then solve the equation (again see Appendix C). Uhlig (1999) shows that the solution to (2.1) takes the form of

$$\hat{y}_t = a_2 \hat{y}_{t-1} + a_3 h_t \qquad (3.1)$$

where a_2 and a_3 are functions of the structural parameters $\hat{a}_1$ and $\hat{a}_2$. Based on our calibration we have $a_2 = 0.7$ and $a_3 = 1.37$ (see Appendix C).

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[8] We adjusted the SARB's official QPM Phillips curve because it employs a 'disaggregated' Phillips curve (services, core goods, electricity, petrol and food inflation equation). However, the impulse responses are not significantly different. To achieve our objective, we needed to make use of one Phillips curve, hence the adjustment.

Using the definition of h_t, and combining (3.1) with the interest rate block of the QPM – equations (2.2) and (2.3) – yields the following first-order difference equation for the ouput gap or the structural mulivariate measure:

$$\hat{y}_t = a_2 \hat{y}_{t-1} - a_3 \hat{a}_3 \left\{ \left(i_t - E_t \pi_{t+1} - \bar{r}_t \right) \right\} + a_3 \left(\hat{a}_4 \hat{lz}_t + \hat{a}_5 \hat{y}_t^F + \hat{a}_6 \hat{tot}_t + \hat{\varepsilon}_t^{yRE} \right) \tag{3.2}$$

3.1 Strict output gap targeting

We now investigate what interest rate settings are required to set the output gap to a zero target. This is known as strict output gap targeting (SOT). Thus, we have

$$0 = a_2 \hat{y}_{t-1} - a_3 \hat{a}_3 \left\{ \left(i_t - E_t \pi_{t+1} - \bar{r}_t \right) \right\} + a_3 \left(\hat{a}_4 \hat{lz}_t + \hat{a}_5 \hat{y}^F + \hat{a}_6 \hat{tot}_t + \hat{\varepsilon}_t^{yRE} \right) \tag{3.3}$$

Call the interest rate that solves (3.3) i_t^{OPT}. Note that ceteris paribus, the output gap only responds to demand-pull factors $\hat{lz}_t, \hat{y}^F$ and $\hat{tot}_t$. The solution is:

$$i_t^{OPT} = E_t \pi_{t+1} + \bar{r}_t + \left(a_3 \hat{a}_3 \right)^{-1} a_2 \hat{y}_{t-1} + \hat{a}_3^{-1} \left(\hat{a}_4 \hat{lz}_t + \hat{a}_5 \hat{y}^F + \hat{a}_6 \hat{tot}_t + \hat{\varepsilon}_t^{yRE} \right) \tag{3.4}$$

Now we must take the zero lower bound into account, so that

$$i_t^* = Max\left(i_t^{OPT}, 0\right) \tag{3.5}$$

In practice, central banks smooth nominal interest rates, so in the end, and in line with the Taylor rule of the QPM, we have

$$i_t^S = f_1 i_{t-1} + \left(1 - f_1\right) i_t^* \text{ where } 0 \le f_1 \le 1 \text{ where } f_1 = 0.8 \tag{3.6}$$

This is the interest rate floor. It is important to realise that both smoothing and the zero lower bound will result in possibly non-zero (off target) output gaps. If we label the actual preferred (ex-post discretionary) policy rate as i_t^{ACT} and if we find actual repo rates below this floor, then it suggests monetary policy has been overly accommodative, that is:

$$i_t^{ACT} < i_t^s \tag{3.7}$$

Due to the zero lower bound and smoothing,[9] (3.3) is no longer always satisfied. The consequence will, ceteris paribus, be the presence of negative output gaps. Even when $f_1 = 1$, the series for the output gap (3.2) will be stationary provided that $\left|a_2\right| < 1$.[10] Note that smoothing and the zero lower bound jeopardise the achievement of zero ouput gaps.

What should be the future (one-quarter ahead) repo rate? Using the forward shift operator, we get:

$$\begin{aligned} 0 &= a_2 \hat{y}_t - a_3 \hat{a}_3 \left\{ \left(i_{t+1} - F\left(E_t \pi_{t+1}\right) - E_t \overline{r_{t+1}} \right) \right\} \\ &+ a_3 \left(\hat{a}_4 E_t \hat{lz}_{t+1} + \hat{a}_5 E_t \hat{y}^F_{t+1} + \hat{a}_6 E_t \hat{tot}_{t+1} + \hat{\varepsilon}^{yRE}_{t+1} \right) \end{aligned} \tag{3.8}$$

This gives us a calibration for the next quarter's repo rate. This lead (forward shift operator) exercise is applicable generally. For example, two-quarters ahead yields:

$$\begin{aligned} 0 &= a_2 \hat{y}_{t+1} - a_3 \hat{a}_3 \left\{ \left(i_{t+2} - F^2\left(E_t \pi_{t+1}\right) - E_t \overline{r_{t+2}} \right) \right\} \\ &+ a_3 \left(\hat{a}_4 E_t \hat{lz}_{t+2} + \hat{a}_5 E_t \hat{y}^F_{t+2} + \hat{a}_6 E_t \hat{tot}_{t+2} + \hat{\varepsilon}^{yRE}_{t+2} \right)^{11} \end{aligned} \tag{3.9}$$

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9 The zero lower bound is required because this is the lowest limit for the interest rate, while the importance of smoothing is to reduce interest rate volatility stemming from the variability of the output gap and inflation.

10 This is the case for the parameter values/calibration of $\hat{a}_1$ and $\hat{a}_2$ in the QPM.

11 This can also be done for the cases of strict and flexible inflation targeting and for more quarters ahead.

3.2 Strict inflation targeting

Now we look at the case where the central bank only cares about inflation outcomes, that is strict inflation targeting (SIT). In what follows we investigate a point target. Setting inflation at target in the Phillips curve we have:

$$\pi^* = b_1 \hat{y}_t + b_3 \pi_{t-1} + j_t \quad (3.10)$$

From the rational expectations solution of the output gap, we get:

$$\pi^* = b_1 \left(a_2 \hat{y}_{t-1} + a_3 h_t \right) + b_3 \pi_{t-1} + j_t \quad (3.11)$$

Using the expression for h_t we get:

$$\pi^* = b_1 \left(a_2 \hat{y}_{t-1} + a_3 \left\{ -\hat{a}_3 \hat{r}_t + \hat{a}_4 \hat{lz}_t + \hat{a}_5 \hat{y}^F + \hat{a}_6 \hat{tot}_t + \hat{\varepsilon}_t^{yRE} \right\} \right) \quad (3.12)$$
$$+ b_3 \pi_{t-1} + j_t$$

When we combine (3.12) with the interest rate block from the QPM (equations (2.2) and (2.3)), we obtain:

$$\pi^* = b_1 \left(a_2 \hat{y}_{t-1} + a_3 \left\{ \begin{array}{l} -\hat{a}_3 \left\{ \left(i_t - E_t \pi_{t+1} - \bar{r}_t \right) \right\} \\ + \hat{a}_4 \hat{lz}_t + \hat{a}_5 \hat{y}^F + \hat{a}_6 \hat{tot}_t + \hat{\varepsilon}_t^{yRE} \end{array} \right\} \right) \quad (3.13)$$
$$+ b_3 \pi_{t-1} + j_t$$

The interest rate that solves the above expression is captured by equation (3.14):

$$i_t^{OPT} = E_t \pi_{t+1} + \bar{r}_t + \left(b_1 a_3 \hat{a}_3 \right)^{-1} \left(b_3 \pi_{t-1} - \pi^* \right)$$
$$+ \left(b_1 a_3 \hat{a}_3 \right)^{-1} \left(\begin{array}{l} b_1 a_2 \hat{y}_{t-1} + b_1 a_3 \hat{a}_4 \hat{lz}_t \\ + b_1 a_3 \hat{a}_5 \hat{y}_t^F + b_1 a_3 \hat{a}_6 \hat{tot}_t + j_t + b_1 a_3 \hat{\varepsilon}_t^{yRE} \end{array} \right) \quad (3.14)$$

Note that, ceteris paribus, the latter now responds to both demand-pull factors ($\hat{lz}_t, \hat{y}^F, \hat{tot}_t$) and cost-push factors (j_t). We expect the optimal nominal repo rate (after the zero lower bound and smoothing) to sit at the ceiling associated with an upper limit of the nominal repo rate.[12]

3.3 Flexible inflation targeting

Flexible inflation targeting is relevant when the central bank pays attention to both the inflation gap (inflation minus target inflation) and the (zero) output gap. So, we have the loss function:

$$L = \left(\hat{y}_t\right)^2 + \lambda_1\left(\pi_t - \pi^*\right)^2 \qquad (3.15)$$

Here, the weight on the inflation gap is λ_1. It can be shown that the corner solutions, that is, ($\lambda_1 = 0$ and $\lambda_1 \to \infty$), result in, respectively, SOT and SIT. We use equal weights for the two gaps since we don't know this parameter. Thus, in terms of calibration we set $\lambda_1 = 1$. Using the Phillips curve we get:

$$\underset{\hat{y}_t}{Min} = \left(\hat{y}_t\right)^2 + \lambda_1\left(b_1\hat{y}_t + b_3\pi_{t-1} + j_t - \pi^*\right)^2 \qquad (3.16)$$

As can be seen above, we use the ouput gap as the indirect control. The solution is

$$\hat{y}_t = -\frac{\lambda_1 b_1}{\left(1+\lambda_1\left(b_1\right)^2\right)}\left(b_3\pi_{t-1} - \pi^*\right) - \frac{\lambda_1 b_1}{\left(1+\lambda_1\left(b_1\right)^2\right)} j_t \qquad (3.17)$$

Again, this can be combined with the IS curve and interest rate block from the QPM. We expect the optimal nominal repo rate (after the zero lower bound and smoothing) to sit in a corridor with the floor being SOT and the ceiling being SIT.

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12 The reason for this is that the limiting cases of the loss function are SIT and SOT. SIT implies a lower output gap than SOT (which implies a zero output gap). Hence SIT gives us the maximum interest rate and SOT gives us the lowest interest rate. In the in-between case of flexible inflation targeting, the associated interest rates must lie within these limiting cases or bounds (floor and ceiling). Thus, in the case of SIT, interest rates sit at the upper bound or ceiling.

4. Empirical results

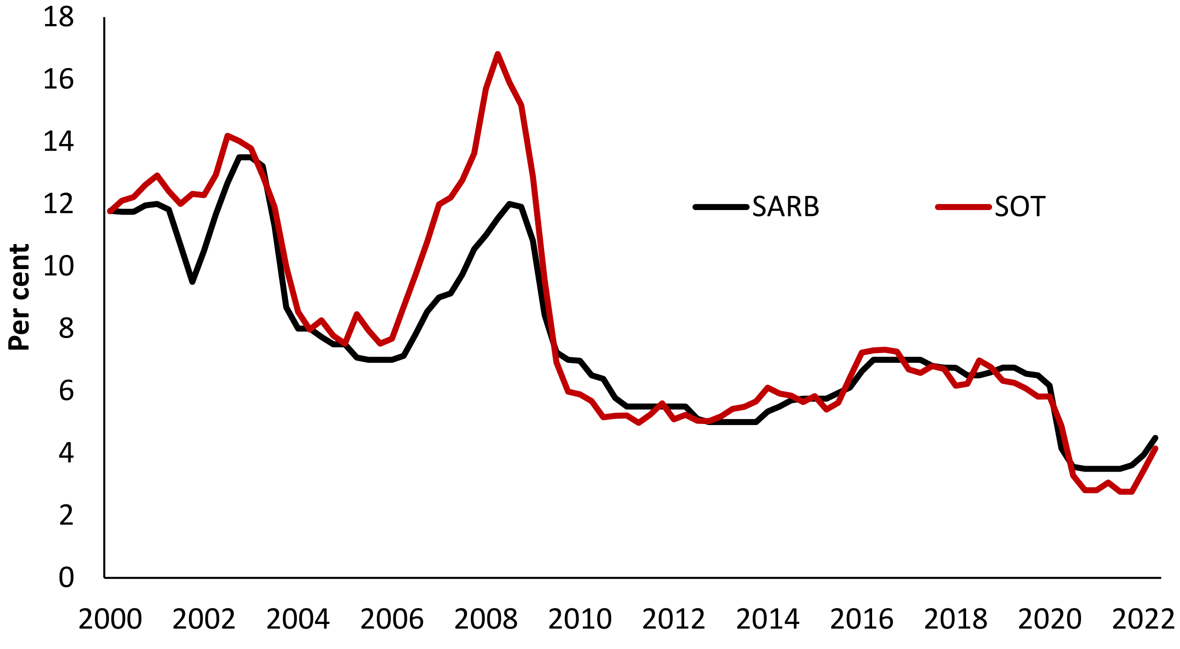
4.1 Data

Our dataset comprises quarterly data from the first quarter of 2000 to the second quarter of 2022. Although there is data available from before 2000, we use post-2000 data to capture the inflation-targeting period. All the data used (including equilibria such as potential GDP and the neutral real interest rate) is the same as the data used in the SARB's current modelling framework. Table A1 in Appendix A shows the data used as inputs in this paper.

4.2 Deviations of actual repo from implied SOT and SIT repo rates

We start our analysis by investigating the monetary policy stance of the SARB since 2000, when it adopted flexible inflation targeting. To assess the SARB's monetary policy stance, we use the SOT and SIT Taylor rule implied repo rate. Figure 2 shows the actual repo and SOT implied repo, which shows a strong relationship between the two, with a correlation coefficient of around 95%. This indicates that the SARB does consider the output gap when making interest rate decisions.

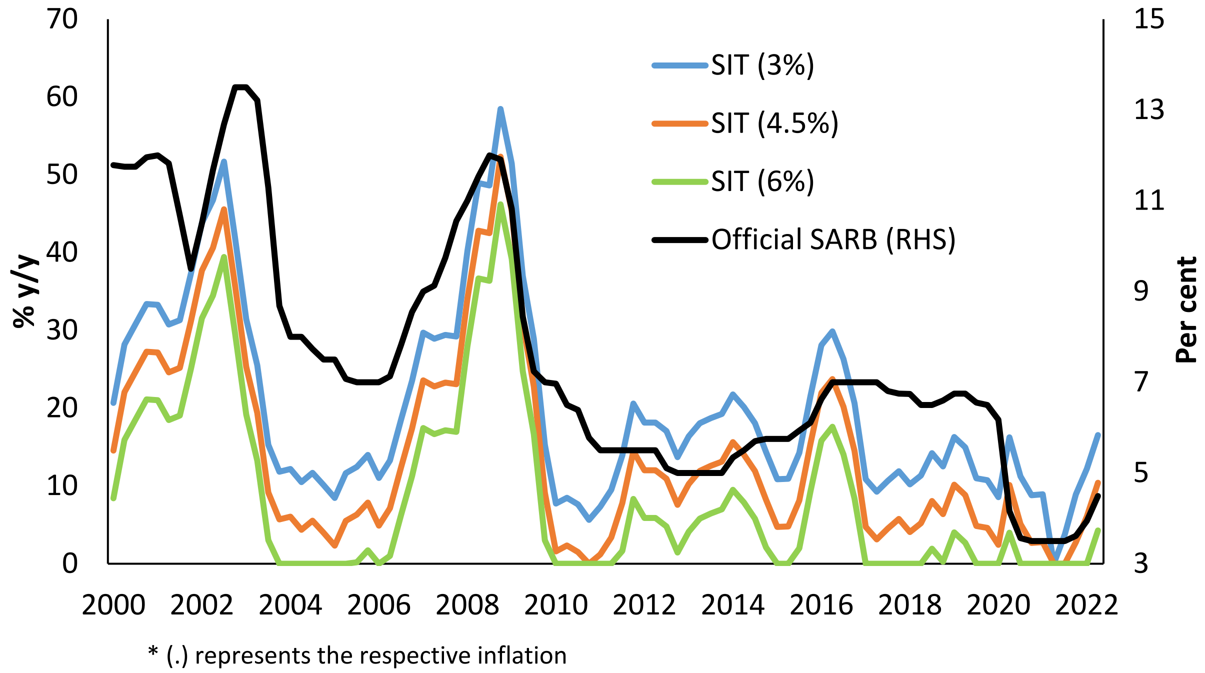
Figure 2: Actual and SOT implied repo rate



Source: SARB and own calculations

Similarly, Figure 3 shows the actual repo rate and SIT implied repo rate (for a 3%, 4.5% and 6% inflation target), which also has a high correlation coefficient of around 75%. This is in line with the fact that the SARB is an inflation-targeting central bank, responding to deviations of inflation from the target. Moreover, it is worth noting that a lower inflation target (3%) has a higher SIT implied repo rate, and vice versa. This is consistent with our priors and the literature (Cecchetti and Rich 2001; Loewald, Makrelov and Pirozhkova 2022). The volatility in the SIT implied repo rate is largely due to the volatility in inflation (see Figure 4). Additionally, the large magnitude in the SIT implied repo rate is because there is no direct link between inflation and interest rates. The relationship is largely indirect in the model – via the output gap (with the output gap having a relatively small coefficient of 0.25 in the Phillips curve). Therefore, for a SIT rule to bring inflation to target relatively quickly, it will require policy to react very aggressively.

Figure 3: Actual and SIT implied repo rate*



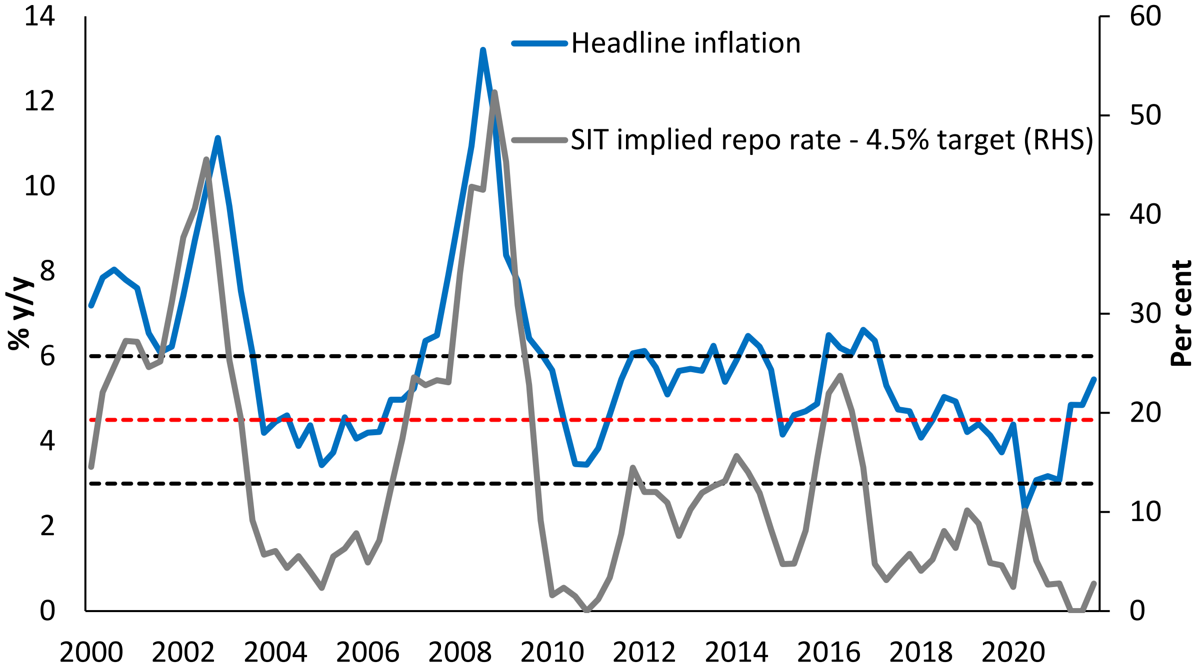
Source: SARB and own calculations

The SOT and SIT implied interest rate are meant to serve as a range for interest rates (i.e., a floor and a ceiling). Therefore, deviation of the actual repo rate from both the SOT and SIT implied repo rate (in the same direction[13]) would indicate whether monetary policy is too restrictive or too accommodative. If the actual rate is above the

13 For example, if the actual repo rate is below (or above) both the SOT and SIT implied repo.

flexible inflation targeting (FIT) implied repo rate, then policymakers probably placed a bigger weight on inflation; whereas if the actual repo rate is below the FIT implied repo rate, then policymakers probably placed a bigger weight on growth or the output gap (see Figure 5). The magnitude of the deviation matters as it would determine the level of accommodativeness or restrictiveness of policy at the time.

Figure 4: Headline inflation and SIT (4.5%)



Source: Statistics South Africa and own calculations

Our methodology suggests that in the 20 years under review, monetary policy has been mostly accommodative. This is consistent with findings by Loewald, Faulkner and Makrelov (2020), and Honohan and Orphanides (2022). We find that between 2000 and 2002 the actual repo was below both the implied SOT and the SIT (6%)[14] implied repo rate. This indicates that monetary policy was too accommodative (i.e., interest rates were not raised high enough). Much higher interest rates at the time would have been more appropriate as inflation was well above the upper end of the target (averaging 7.9%) and the output gap was positive (averaging 0.8%). This finding is supported by the SARB's October 2002 *Monetary Policy Review*, which stated that "monetary conditions are not as tight as suggested by looking solely at the level of

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[14] See Figure 7, which shows the deviation of the actual repo rate from the SIT (6%) implied repo rate. We compare actual to SIT (6%) during this period because inflation was for most of the period close to or above the 6% target and the SARB was not communicating a preference for inflation at 4.5% yet.

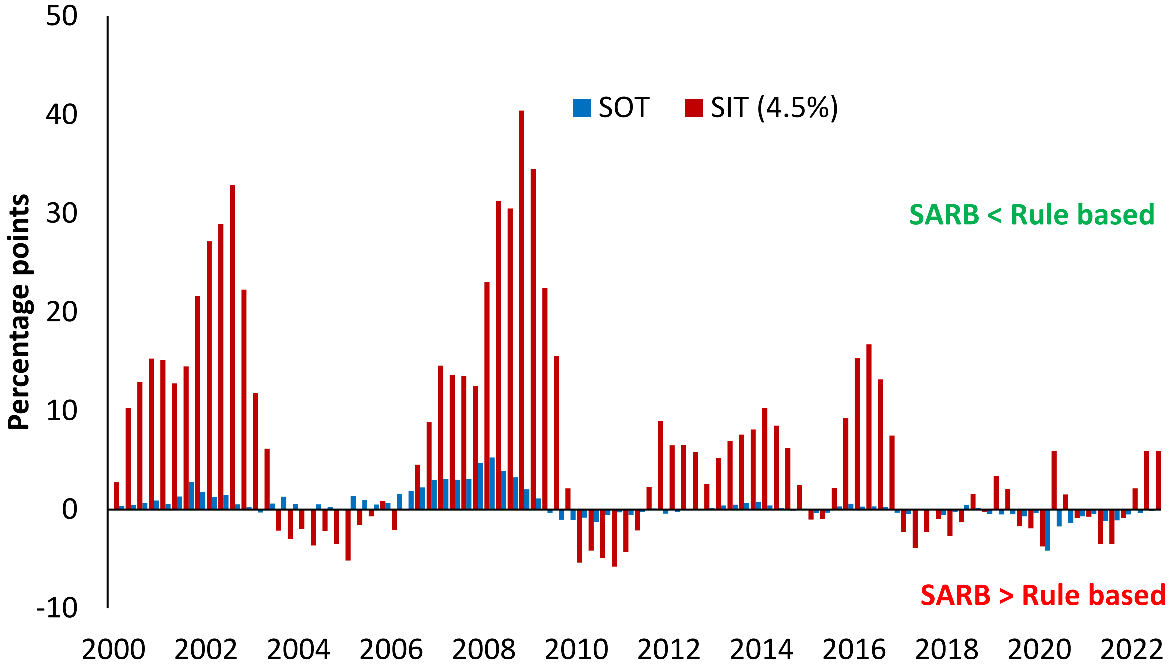
interest rates" (SARB, *Monetary Policy Review*, October 2002, p. 20). According to the SARB, monetary policy responses to inflation shocks require a rise in real interest rates. However, between 2000 and 2002, inflation was above 6% for most of the period, but real interest rates did not rise as much. In fact, they were lower in 2002, and this is the period where inflation was even more elevated (SARB, *Monetary Policy Review*, October 2002). This indicates that interest rates were probably not raised high enough given the prevailing conditions. However, our results are contrary to findings by Coco and Viegi (2020), who suggest that the monetary policy stance was tight in the period 2000 to 2002. This could largely be explained by differences in the estimated unobservable variables such as the neutral real interest rate and potential GDP.[15]

Similarly, between 2006 and early 2009 (before the global financial crisis), monetary policy was much more accommodative. Basically, the policy rate was not raised enough, despite high inflation[16] (which exceeded the upper end of the target, averaging 7.8% and hitting double digits in 2008Q2–2008Q4) and a very positive output gap (averaging 2.7% over the period). Even with an inflation target of 6%, the observed policy rate would still be interpreted as too accommodative during that period (see Figure 7). One of the reasons that the actual repo rate was not strong/aggressive enough was that there were significant underestimates in forecasts of GDP and inflation (Honohan and Orphanides 2022). These forecasts are among the inputs considered by the Monetary Policy Committee (MPC) when making policy decisions, hence this could have contributed to policy being less reactive.

15 The study by Coco and Viegi (2020) does test for the robustness of the results using the SARB's unobservable variables, and their results are broadly unchanged. However, these have been revised (due to revisions in the historical GDP data). For example, the output gap is now estimated to have been above positive one in the period 2000 to 2002. However, before the revisions, the output gap was estimated to be closer to zero.

16 Inflation drivers during this period include the effects of elevated international oil prices and a significant depreciation of the rand.

Figure 5: Deviation of actual repo from the SOT and SIT rule

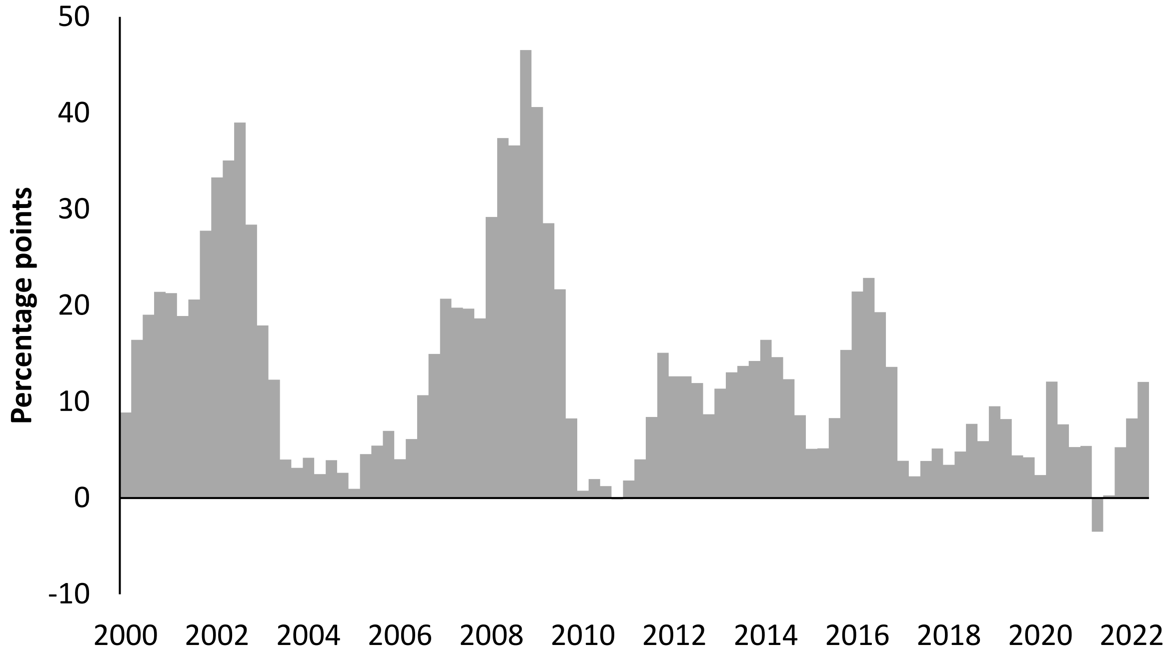


Source: SARB and own calculations

However, as inflation began to slow around 2009, almost reaching the bottom of the target range in September 2010, the actual policy rate was reduced significantly to counter the global recession. The output gap was also negative due to the significant deterioration in global demand (or economic growth). This was consistent with the SOT and SIT implied repo rate. However, the actual repo rate was slightly above the SIT and SOT implied repo rate in late 2009 to 2010, suggesting that the repo rate was not lowered fast enough (with slightly tighter monetary policy, even with a 6% inflation target). However, it is worth noting that the deviation is not as large as the 2000 to 2002 and 2006 to early 2009 periods (discussed above) when the repo rate was not raised high enough. Our results are consistent with those of Honohan and Orphanides (2022), who also found that the repo rate in South Africa was not high enough in 2006–2008 and not low enough in 2010.

From 2012 to 2015, the actual repo was somewhat accommodative (slightly below both the SIT (4.5%) and SOT implied repo rate). However, this is because our SIT in Figure 5 is estimated using an inflation target of 4.5%. The deviation from the SIT with a 6% inflation target shows that the actual repo rate was within the SIT (6%) and SOT implied repo rate benchmark. This suggests that the midpoint of the target range was not yet seen as the preferred target for inflation at the time, as it was probably perceived to be too low.

Figure 6: Deviation of actual repo from SIT (3%)

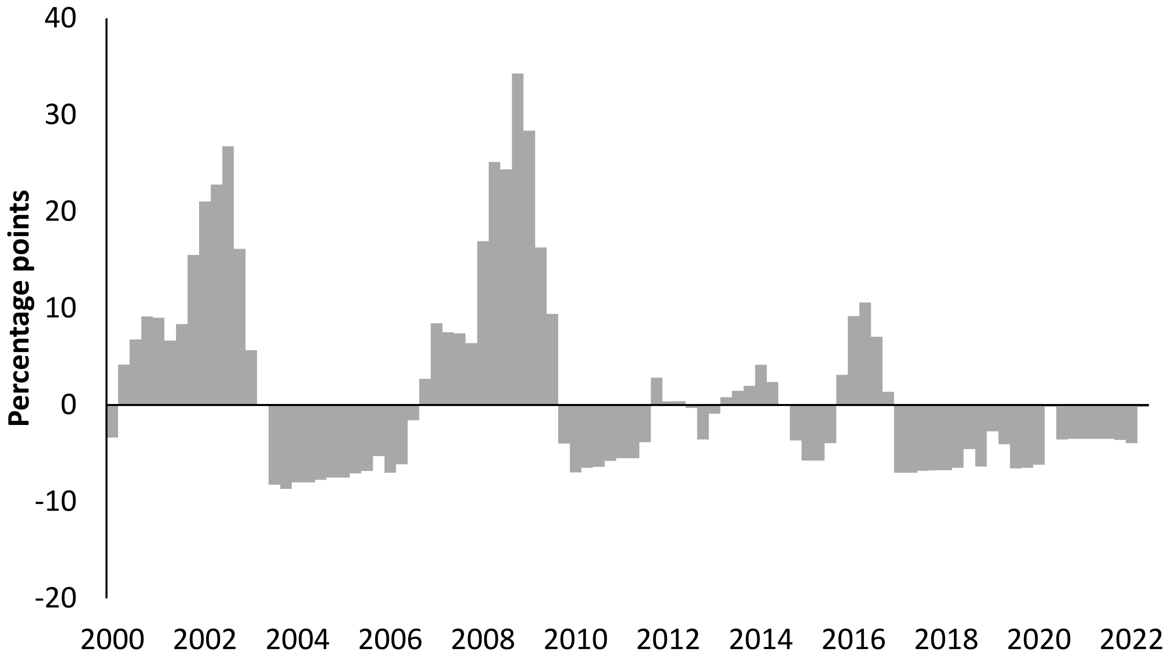


Source: Own calculations

Interestingly, using a 3% inflation target, our methodology shows that the actual repo rate has been below the SIT (3%) throughout the sample. This is because inflation was relatively high over the sample period (well above 3%) and therefore the SARB's interest rate decisions were probably not aiming to steer inflation to 3% as that would have required policy to be very restrictive (see Figure 6). This is confirmed by Klein (2012), who finds that the implicit inflation target tends to drift towards the upper end of the target range (6%).

In 2016, the actual repo was again below both the SIT (4.5% and 6%) and SOT implied repo rate. This suggests that the repo rate was too accommodative (not high enough), as inflation was above the upper end of the target range throughout 2016 due to the effects of drought (which raised food inflation to an average of 10.5% in 2016). The MPC probably interpreted this as more of a shock of supply rather than demand, hence the repo rate was not raised high enough (the policy response was not aggressive enough).

Figure 7: Deviation of actual repo from SIT (6%)



Source: Own calculations

Between 2017 and 2019 (before the COVID-19 shock), the actual repo rate was very close to both the SOT and SIT (4.5%) implied repo rate. This indicates that SARB was committed to its preference to keep inflation at the midpoint of the target range (4.5%). This is confirmed by looking at the SIT (6%) implied rate, which would suggest that monetary policy was tighter during this period. This accords with the fact that around 2017 the SARB started communicating its preference for inflation to be at 4.5%. In most MPC statements from November 2017, the SARB emphasised its preference for inflation to be at 4.5% and, according to our methodology, the interest rate decisions made were also consistent with this preference. Interestingly, inflation was also closer to 4.5% for most of this period and inflation expectations (which were previously sticky at the upper end of the target) converged towards the midpoint of the target. Although there were some positive shocks which contributed to this, we cannot rule out the view that effective monetary policy also played a role.

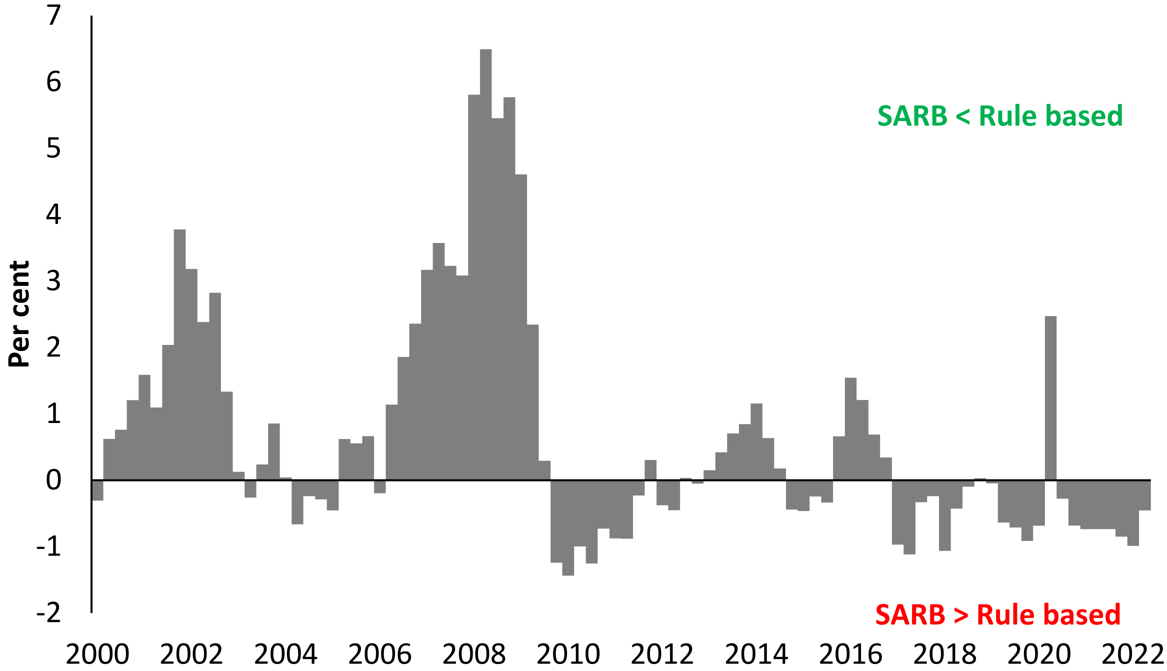
To illustrate the SARB's commitment to keeping inflation closer to 4.5%, in the November 2018 MPC the SARB hiked the repo rate to 6.75%. The SARB did this even though inflation was around 5% and projected to be at 5.4% two years ahead. Before 2017, if inflation was around these levels, the repo rate would probably have not been raised. The MPC justified the 2018 repo rate hike by noting that "delaying the adjustment could cause inflation expectations to become entrenched at higher levels and thus contribute to second round effects, which would require an even stronger monetary policy response in the future" (SARB, MPC statement, November 2018, p. 5).

During the COVID-19 pandemic period, the SARB sharply reduced the repo rate to a historic low level of 3.75%. This was an appropriate decision given the collapse in domestic and global output gaps as well as low inflation in 2020. However, our methodology suggests that the actual repo rate should have been slightly lower than it was (as it was above both the SOT and SIT implied repo rate). The SARB probably did not lower the repo rate further as it was cognisant of the inflation risk ahead given the significantly weak exchange rate, gradual increase in demand and the supply bottlenecks caused by lockdown restrictions.

4.3 Deviations of the actual repo from implied FIT repo rate (50-50 weight)

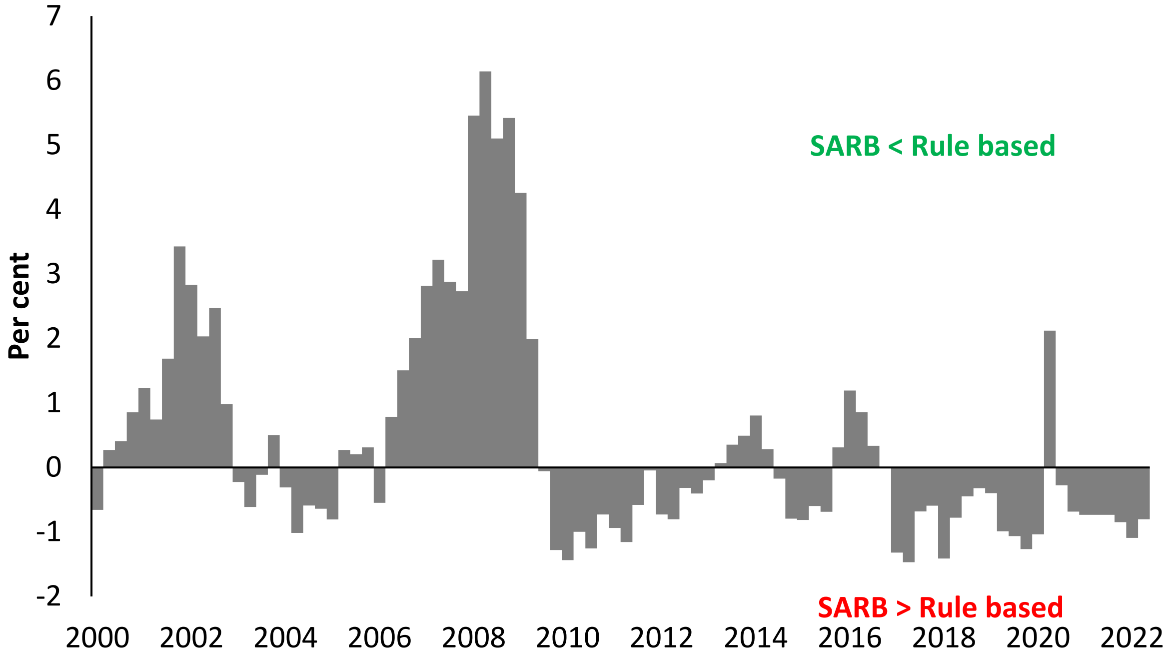
In this section, we calculate the historical implied repo rate using an FIT rule, with an equal weight between the output gap and inflation (see section 3.3). We then compare this repo rate profile to the actual repo rate. If the actual is above the FIT implied repo rate, then policymakers probably placed a bigger weight on inflation, and if the actual repo is below the FIT implied repo rate, then policymakers probably placed a bigger weight on growth/the output gap. Our results suggest that in periods where we deem policy was more accommodative, a larger weight was placed on growth, while in periods where we deem policy was somewhat restrictive, a larger weight was placed on inflation. Overall, our methodology suggests that the SARB has mostly placed a bigger weight on growth/the output gap relative to inflation. This finding is consistent with findings by Ellyne and Veller (2011) and Klein (2012), who find that the SARB mostly placed a greater weight on output relative to inflation. However, since the SARB's adoption of the QPM in 2017, policymakers have placed a bigger weight on inflation than growth, and rightfully so. The exception was during the early stages of the COVID-19 pandemic, when the negative effects of COVID-19 on South Africa's growth/output gap resulted in a large cut in the repo rate to help support growth during the time (see Figures 8 and 9).

Figure 8: Deviation of actual repo rate from FIT (4.5%)



Source: SARB and own calculations

Figure 9: Deviation of actual repo rate from FIT (6%)



Source: SARB and own calculations

4.3 Impulse response functions

In this section, we will include the SOT and SIT rules in the SARB's main QPM.[17] This will allow us to test if the two rules[18] derived from our small two equations model could serve their purpose in the main model that has more channels or equations. We will consider the model's response to an output gap shock and an inflation shock.

4.3.1 Output gap shock

Figure 10 plots the impulse response functions (IRFs) from a 1% shock to the output gap. The SOT rule shows the repo rate responding earlier and more aggressively relative to the QPM and the SIT rule. This indicates that the SOT rule does indeed target an output gap of zero (as it is calibrated to do so). When the output gap closes, the SOT repo rate decreases as well, despite inflation being higher and even remaining above target for longer relative to the SIT and QPM. Additionally, the SIT policy response is somewhat delayed because it is only responding to the higher inflation which is pushed up by the higher output gap shock with a lag effect.

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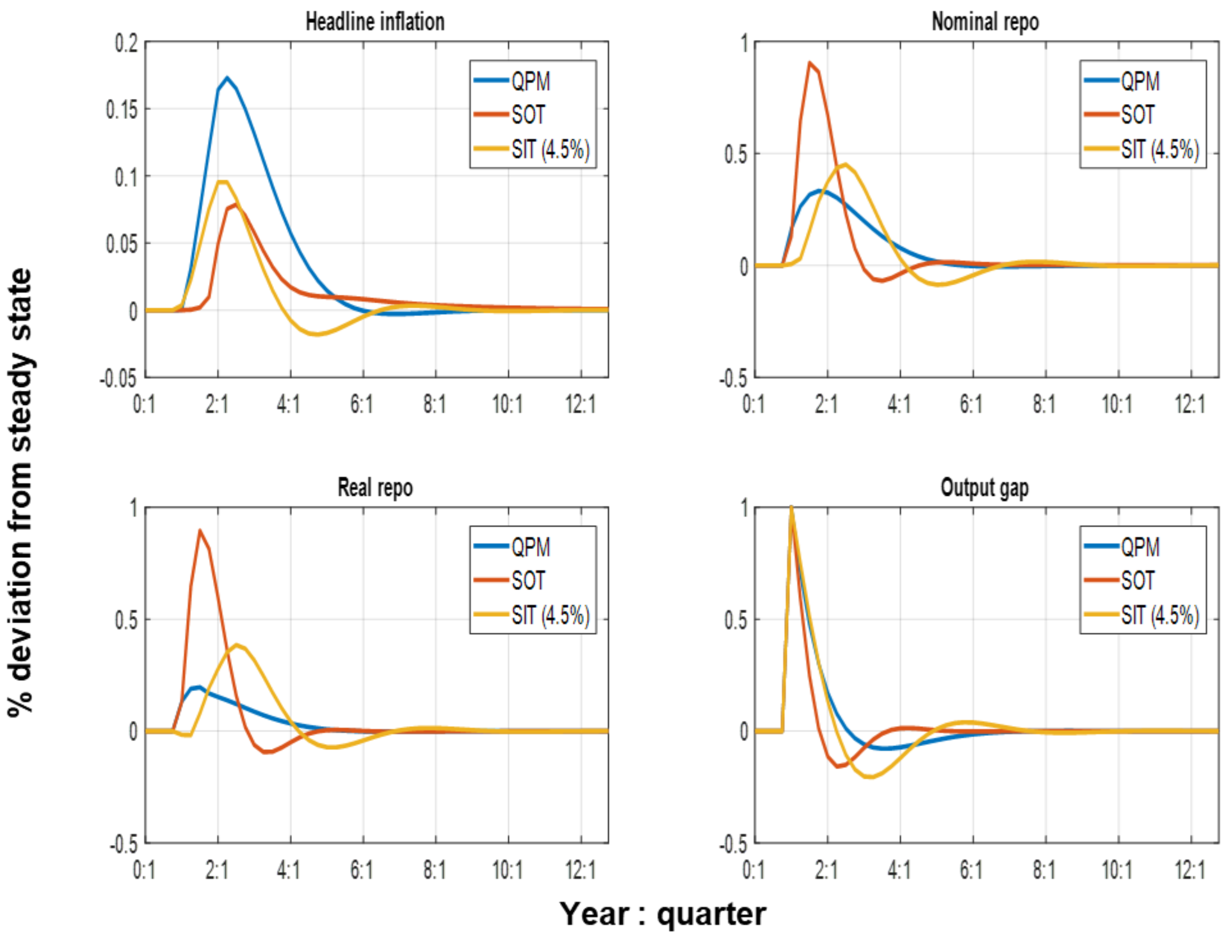
17 For more details on the response of the QPM to these shocks, see Botha et al. (2017).

18 The SIT and SOT were solved using a small version of the main QPM model (two equations).

Figure 10: 1% shock on the output gap

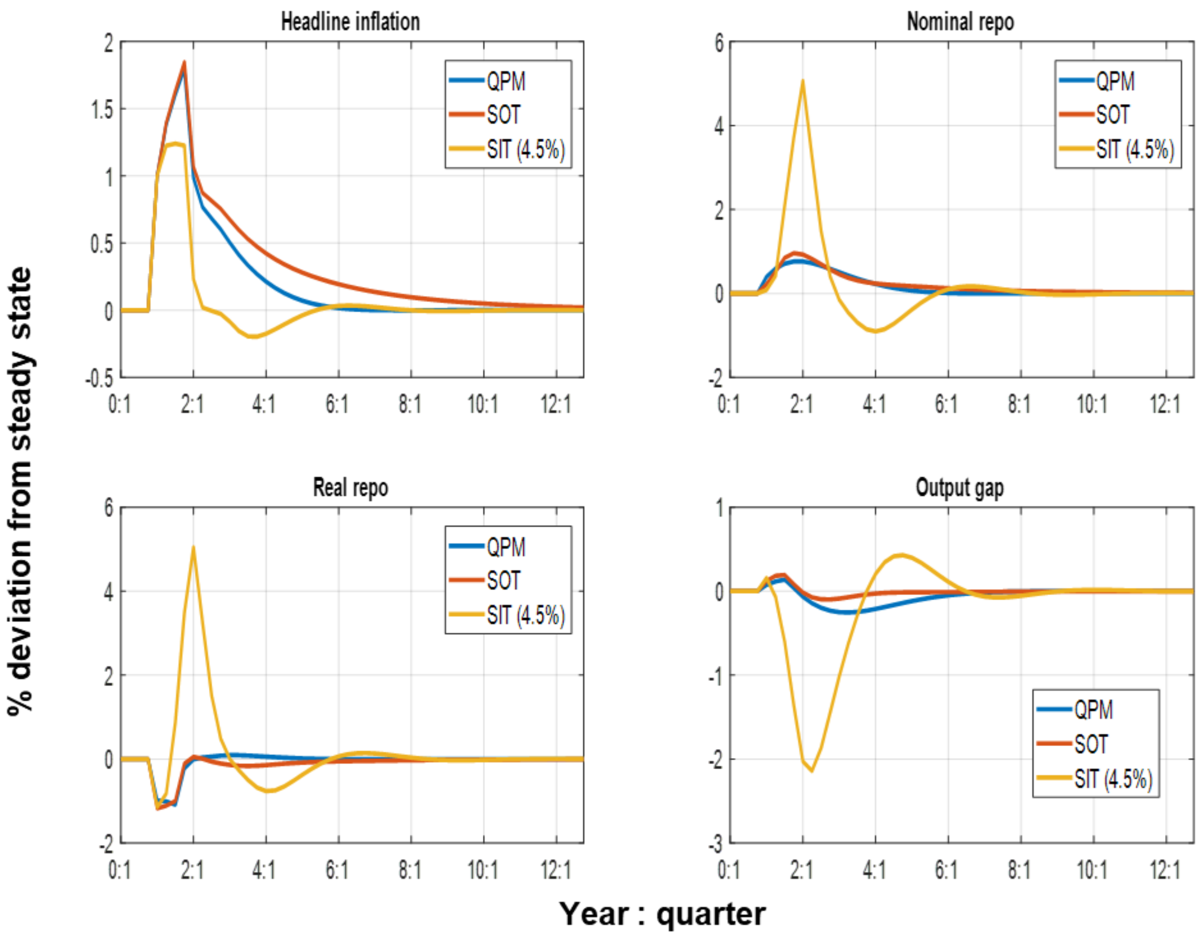


Source: SARB and own calculations

4.3.2 Inflation shock

Figure 11 plots the IRFs from a 1% shock to headline inflation. The SIT now responds more aggressively relative to the SOT rule. The SIT response then ensures that inflation converges to target much faster than the SOT rule. The SOT rule again aims to close the output gap faster despite inflation remaining above the target for longer. This indicates that our calibration is consistent with our prior.

Figure 11: 1% inflation shock

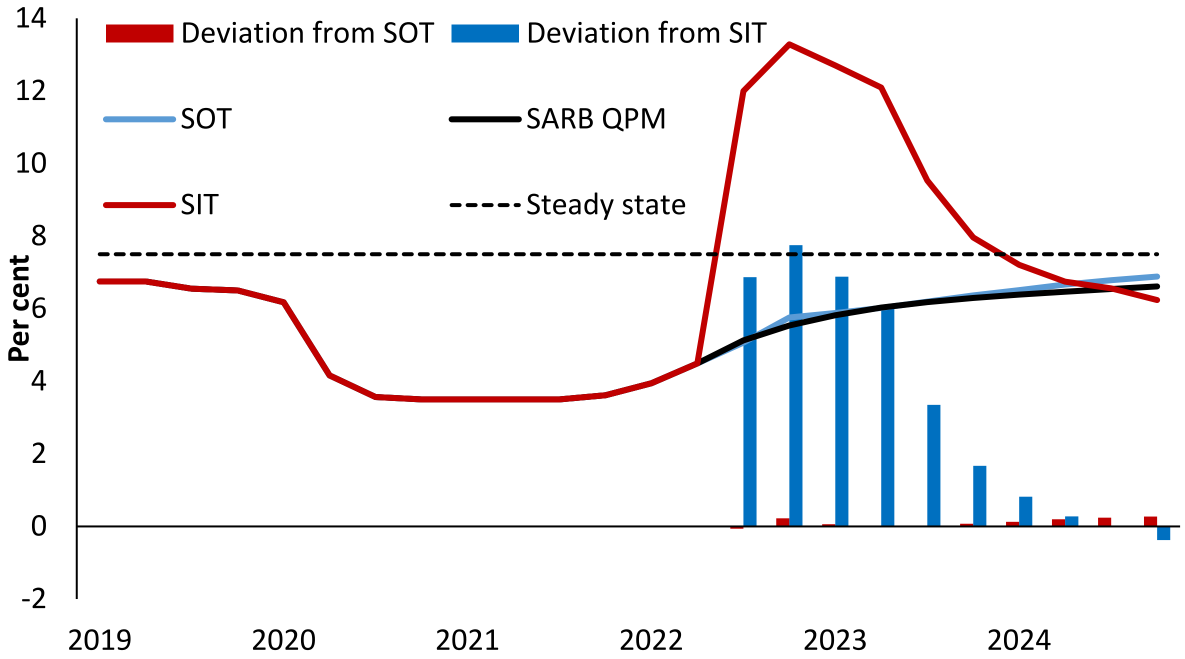


Source: SARB and own calculations

4.4 Forecasting

We now use these rules to forecast with the main SARB QPM. This will be useful to benchmark the SARB's repo path suggested by the QPM over the forecast period. We use the July 2022 SARB MPC dataset and information. First, we forecast the SOT and SIT repo rate path by exogenising the drivers or other input variables. Basically, the path of inflation, output gap and other variables will be equal to those which are forecasted in the July 2022 MPC baseline. We also do an endogenous forecast which allows these variables to vary across the different Taylor rules (or interest rate profiles).

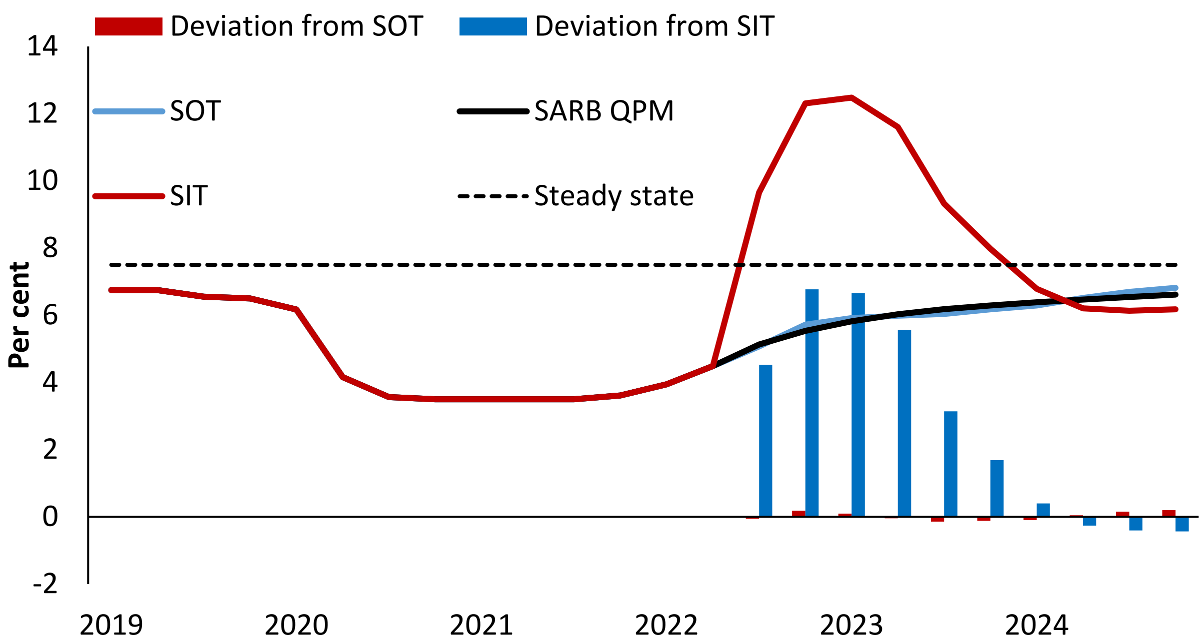
Figure 12: Repo rate forecast – exogenous drivers



Source: SARB and own calculations

Our results show that a SIT central bank would raise the repo rate significantly in the near term (see Figure 12). This is reasonable given that inflation is elevated in the wake of the Russia-Ukraine war. Over the medium term, the repo rate normalises as inflation converges to the midpoint of the target. On the other hand, the SOT implied repo rate is quite close to the SARB's QPM path. It gradually increases as the output gap closes over the forecast horizon.

Figure 13: Repo rate forecast – endogenous drivers

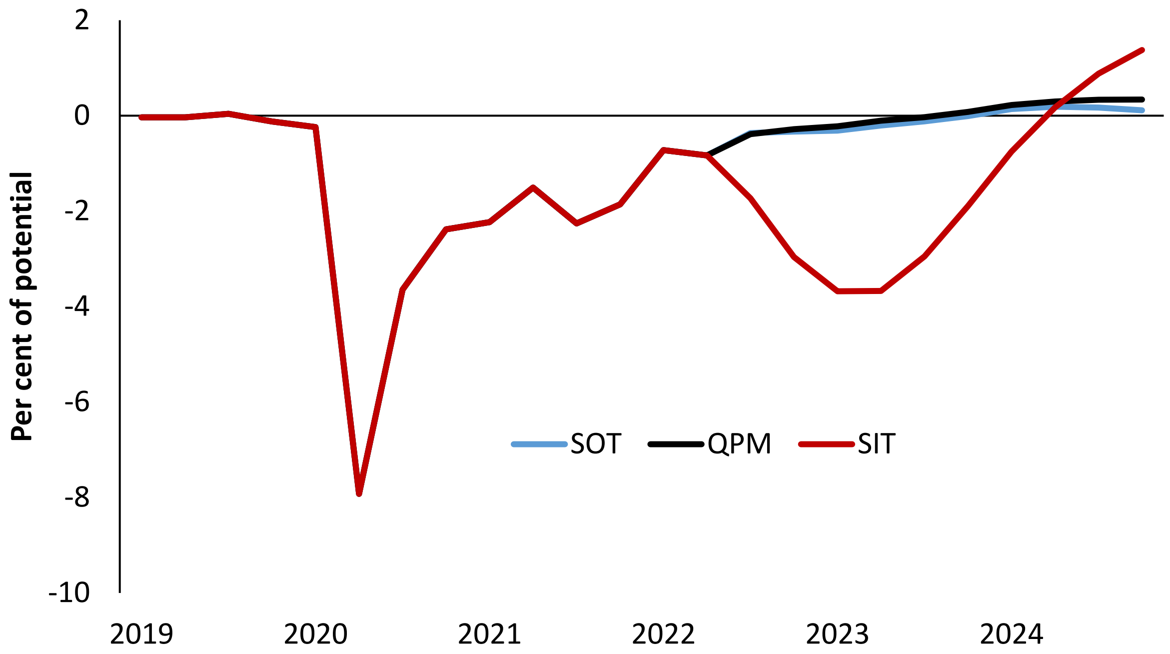


Source: SARB and own calculations

If we allow the other drivers to change as the repo rate changes, the results are not much different to those discussed above (see Figure 13). However, there are slight differences over the medium term, as inflation and the output gap vary across the three rules. Figure 14 shows the output gap forecast. The SOT path ensures that the output gap closes as specified, while the SIT has a much wider output gap as it aims to bring inflation back to target irrespective of a negative output gap. Figure 15 shows that the SIT repo rate path ensures that inflation gets to target much faster relative to the other rules.

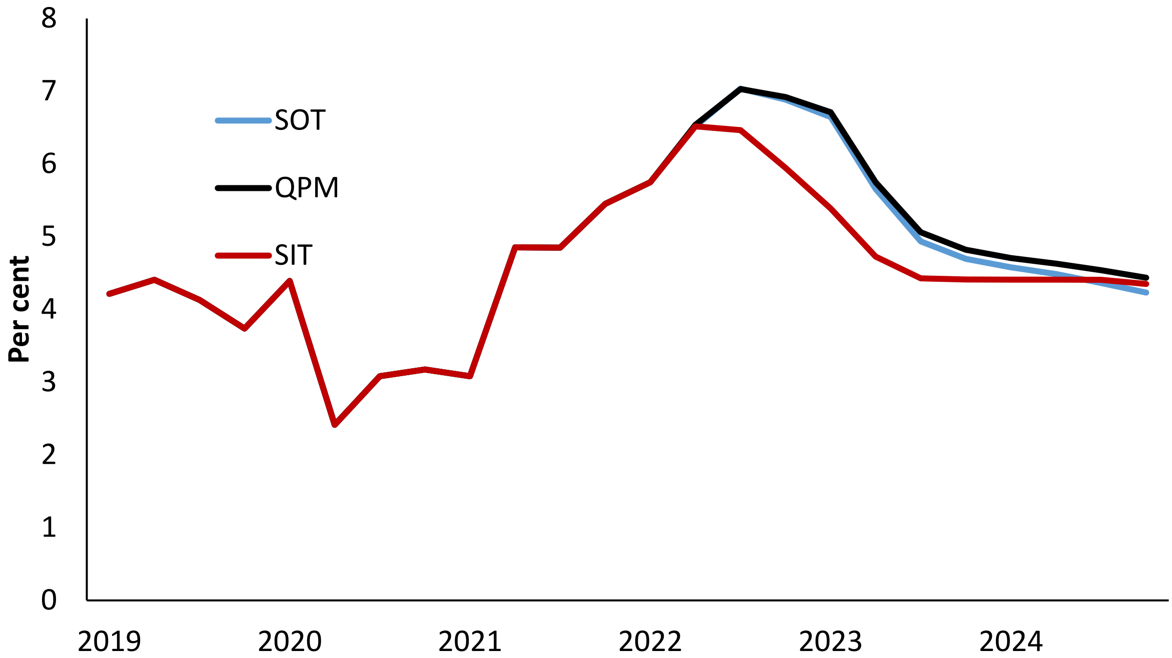
In both Figures 12 and 13, the SARB's QPM repo path is relatively close to the SOT implied repo path over the forecast period. This is because the SARB's QPM Taylor rule was designed to mimic the MPC's historical adjustments to the repo rate (Botha et al. 2017). However, our results suggest that the actual historical repo rate has largely placed more weight on output and has been more accommodative than restrictive. Therefore, since the QPM Taylor rule was built to be closer to the actual historical repo rate, it will also be closer to a Taylor rule that places more weight on output (because our finding in section 4.3 suggests that historically, policymakers probably placed a bigger weight on output) – in this case, that is the SOT rule. Hence the QPM and SOT repo paths are relatively close. We suggest a recalibration of the SARB's QPM Taylor rule that doesn't necessarily aim to be closer to the historical actual repo rate, as this will also include past policy errors/behaviour as well. We believe it would be more appropriate to calibrate the Taylor rule so that it ensures that inflation reverts to target and the output gap closes, putting more weight on inflation (as the SARB is an inflation-targeting central bank).

Figure 14: Output gap forecast



Source: SARB and own calculations

Figure 15: Headline inflation forecast



Source: SARB and own calculations

5. Summary and suggestions for further research

In this paper we built a small macro model that combines the rational expectations version of the SARB's QPM IS curve with a hybrid Phillips curve. We infer optimal monetary policy rules in terms of interest rate gaps (as defined by the QPM) – and hence repo rates – that implement strict inflation targeting, flexible inflation targeting, and strict output gap targeting augmented with interest rate smoothing and the zero lower bound. Among many empirical results we find that between 2000 and 2002 the actual repo rate was below both the strict output gap targeting and strict inflation targeting implied repo rate. This implies that monetary policy was too accommodative. Similarly, between 2006 and early 2009 (before the global financial crisis), monetary policy appears to have been too accommodative. We find no evidence of episodes where monetary policy was too aggressive (even with a 6% inflation target). When comparing the historically implied repo rate using an FIT rule – with an equal weight on the output gap and inflation – we find that, since the adoption of the QPM in 2017, policymakers seem to have placed a bigger weight on inflation than the output gap. Moreover, we suggest a recalibration of the SARB's QPM Taylor rule to a rule that doesn't follow the historical actual repo rate, as this will also include past policy errors and it doesn't consider the different preferences of the MPC.[19]

For further research, we suggest incorporating monetary policy control errors or only partially observed shocks in this model. The idea is that the central bank can respond to demand or supply shocks but does not 'see' the entire shock. It can only respond to the part of the shock that is in its information set. This is due to timing: the central bank sets policy and then the part of the shock arrives that it could not observe. Thus, the central bank has imperfect information and actual inflation and output gaps would not be equal to what is implied by its optimal policy rules.

[19] For example, the MPC now prefers inflation at 4.5%, which is different from their preference before 2017.

Appendices

A. Data

Table A1 Data

Variable	Source
Headline CPI	Statistics South Africa
Average nominal wages	Statistics South Africa
Foreign consumer prices (Weighted US, euro area, Japan)	iGPM Network
Bureau for Economic Research two-year ahead inflation expectations	Bureau for Economic Research
Commodity price index	SARB
International food price index	Haver
Brent crude oil price index	Bloomberg
Real GDP	Statistics South Africa
Foreign GDP	iGPM Network
Repurchase rate	SARB
Nominal exchange rate (Weighted cross-rates with US dollar, euro and yen)	SARB
Country risk premium (EMBI+SA)	JPMorgan
Foreign policy rate	iGPM Network
Domestic potential growth	SARB
Domestic output gap	SARB
Foreign output gap	iGPM Network
Commodity price gap/terms of trade gap	SARB
Domestic NRIR	SARB
Real unit labour cost gap	SARB

B. Variable names and naming conventions

x Log-level variable multiplied by 100 (except in the case of interest rates and the like).

x^* Foreign (trading partner's) variable (with the exception of the inflation target and optimal interest rates)

$\bar{x}$ Equilibrium variable of x

$\hat{x}$ Gap variable of x (deviation from equilibrium)

More specifically,

$\hat{y}_t$ Output gap

$\hat{lz}_t$ Real exchange rate gap from equilibrium (percentage deviation)[20]

$\hat{y}_t^F$ Foreign output gap[21]

$\hat{tot}_t$ Commodity prices (labelled terms of trade)

$\hat{r}_t$ Short-term real interest rate gap

r_t Ex-ante real repo rate

$\bar{r}_t$ Exogenous equilibrium real repo or neutral short-term real repo rate time series (steady stated calibrated at 2.5% in the QPM)

i_t Nominal repo rate (policy instrument or control variable)

$\bar{i}_t$ Exogenous equilibrium nominal repo = $\bar{r}_t + \pi^*$, where π^* is the inflation target (set at 4.5% in the QPM)

$E_t\pi_{t+1}$ Inflation expectations (expectations at time t for inflation at time $t+1$)

π_t Inflation rate

$E_t\pi_{t+2}$ Two-year ahead inflation expectations as surveyed by the Bureau of Economic Research

π_t^F Foreign inflation summarising external price pressures from the inflation rate of imported final goods (proxied by foreign producer price inflation in rand terms)

20 Botha et al. (2020, p. 6) point out that the SARB's exchange rate equilibrium is estimated using a filtering approach and tuned to be in line with a suite of satellite models.

21 The potential growth estimate for the foreign sector is calculated from the Global Projection Model (GPM) as the export weighted average for the United States, euro area, Japan, China and other advanced and emerging market economies. For more details see Botha et al. (2017), p. 22.

$\hat{rulc}_t$ The real unit labour cost gap which is based on the cyclical deviation in real wages from labour productivity and is intended to capture the impact of labour market slack

C. The rational expectations solution

C.1 Stability and uniqueness

Rewrite (2.1) as

$$E_t \hat{y}_{t+1} = \left(\hat{a}_1\right)^{-1} \hat{y}_t - \hat{a}_2 \left(\hat{a}_1\right)^{-1} \hat{y}_{t-1} - \left(\hat{a}_1\right)^{-1} h_t \qquad \text{(C.1)}$$

Define $x_{1t+1} \equiv \hat{y}_t$ (C.2)

Substitute (C.2) in (C1):

$$E_t \hat{y}_{t+1} = \left(\hat{a}_1\right)^{-1} \hat{y}_t - \hat{a}_2 \left(\hat{a}_1\right)^{-1} x_{1t} - \left(\hat{a}_1\right)^{-1} h_t \qquad \text{(C.3)}$$

Writing (C.2) and (C.3) in matrix notation we get

$$\begin{bmatrix} x_{1t+1} \\ E_t \hat{y}_{t+1} \end{bmatrix} = A \begin{bmatrix} \hat{y}_t \\ x_{1t} \end{bmatrix} - \left(\hat{a}_1\right)^{-1} h_t \qquad \text{(C.4)}$$

where $A = \begin{bmatrix} 1 & 0 \\ \left(\hat{a}_1\right)^{-1} & -\hat{a}_2 \left(\hat{a}_1\right)^{-1} \end{bmatrix}$

Blanchard and Kahn (1980):

PROPOSITION 1: If $\overline{m} = m$, if the number of eigenvalues of A outside the unit circle is equal to the number of non-predetermined variables, there exists a unique solution. We get the characteristic equation

$$\begin{vmatrix} 1-r & 0 \\ \left(\hat{a}_1\right)^{-1} & -\hat{a}_2\left(\hat{a}_1\right)^{-1}-r \end{vmatrix} = \left(1-r\right)\left(-\hat{a}_2\left(\hat{a}_1\right)^{-1}-r\right)=0 \qquad \text{(C.5)}$$

Using the parameter values of the QPM we have $\hat{a}_1 = 0.15$ and $\hat{a}_2 = 0.70$.

Then it can be shown that the absolute values of the roots (eigenvalues) are 1 and $\left|-4.67\right| = 4.67$.

This means the model (IS equation) is saddle path stable and we have a unique rational expectations equilibrium.

C.2 Solution

Here we use the method of undetermined coefficients as explained by Uhlig (1999). First we rewrite (2.1) as the system:

$$E_t \hat{y}_{t+1} = \left(\hat{a}_1\right)^{-1} \hat{y}_t - \hat{a}_2\left(\hat{a}_1\right)^{-1} \hat{y}_{t-1} - \left(\hat{a}_1\right)^{-1} h_{t+1} \qquad \text{(C.6)}$$

$$h_{t+1} = h_t$$

We conjecture the solution of (2.1) to be

$$\hat{y}_t = a_2 \hat{y}_{t-1} + a_3 h_t \qquad \text{(C.7)}$$

Leading (C.7) and taking expectations at time t we get

$$E_t \hat{y}_{t+1} = a_2 \hat{y}_t + a_3 h_{t+1} = a_2 \hat{y}_t + a_3 h_t \qquad \text{(C.8)}$$

Substituting (C.8) in (C.7) we get

$$\hat{y}_t = -\frac{\left(\hat{a}_2 \hat{a}_1^{-1}\right)}{\left(a_2 - \hat{a}_1^{-1}\right)} \hat{y}_{t-1} - \frac{\left(\hat{a}_1^{-1} + a_3\right)}{\left(a_2 - \hat{a}_1^{-1}\right)} h_t \tag{C.9}$$

Equalising coefficients on (C.7) and (C.9) we get

$$a_2 = -\frac{\left(\hat{a}_2 \hat{a}_1^{-1}\right)}{\left(a_2 - \hat{a}_1^{-1}\right)}, \; a_3 = -\frac{\left(\hat{a}_1^{-1} + a_3\right)}{\left(a_2 - \hat{a}_1^{-1}\right)} \tag{C.10}$$

The first expression can be rewritten as a simple quadratic in a_2;

$$\left(a_2\right)^2 - \left(\hat{a}_1\right) a_2 + \hat{a}_2 \left(\hat{a}_1\right) = 0 \tag{C.11}$$

The roots are:

$$a_{21(2)} = \frac{\hat{a}_1^{-1} \pm \sqrt{\left(\hat{a}_1^{-2}\right) - 4 \hat{a}_2 \hat{a}_1^{-1}}}{2} \tag{C.12}$$

Using the parameter values of the QPM, that is, $\hat{a}_2 = 0.15$ and $\hat{a}_2 = 0.70$, we choose the smaller root which implies that $a_2 = 0.79$. For a_3 we get:

$$a_3 = -\frac{-\left(\hat{a}_1\right)^{-1}}{\left(a_2 - \left(\hat{a}_1\right)^{-1} + 1\right)} \tag{C.13}$$

Given the above calibration, this then implies that $a_3 = 1.37$.

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